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High-frequency Radar Cross Section (RCS) Approximation of a Thin Dielectric Spherical Shell

by Geoffrey H. Goldman and Frank J. Crowne

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Geoffrey H. Goldman and Frank J. Crowne
Sensors and Electron Devices Directorate, ARL

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14. ABSTRACT
The high-frequency radar cross section (RCS) of a thin-filmed dielectric sphere in the far-field was approximated using a model based on two thin parallel dielectric plates. The results were compared to a solution developed by Andreasen based upon simplified boundary conditions. The models were used to estimate the RCS of a balloon with a diameter of 1 m. There was good agreement between the two methods. The RCS of the balloon was much smaller than an identically sized conducting sphere.

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RCS, balloon, shell, sphere

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1. Introduction

There is interest in estimating the radar cross section (RCS) of a small balloon in the far-field. The balloon can be modeled as a thin dielectric spherical shell. Accurate approximations have been derived, but they are computationally complex.[1] A simpler solution is to model the balloon as two thin parallel dielectric plates. The reflection coefficients of the plates can be estimated using the Fresnel equations. The RCS can be calculated using algebra and the theory of superposition.

2. Theory

The high frequency RCS of a thin dielectric spherical shell in the far-field of a source is calculated using a simple model and classical results from electromagnetic theory. The spherical shell is modeled as two thin dielectric plates separated by the diameter of the sphere. A visualization of the model is shown in figure 1. The antenna transmits electromagnetic radiation towards the two plates and then receives the returned signal. The index of refraction of the air is n_1 and the index of refraction of the plates is n_2. The inner radius of the sphere is r and the thickness of the dielectric wall is d.

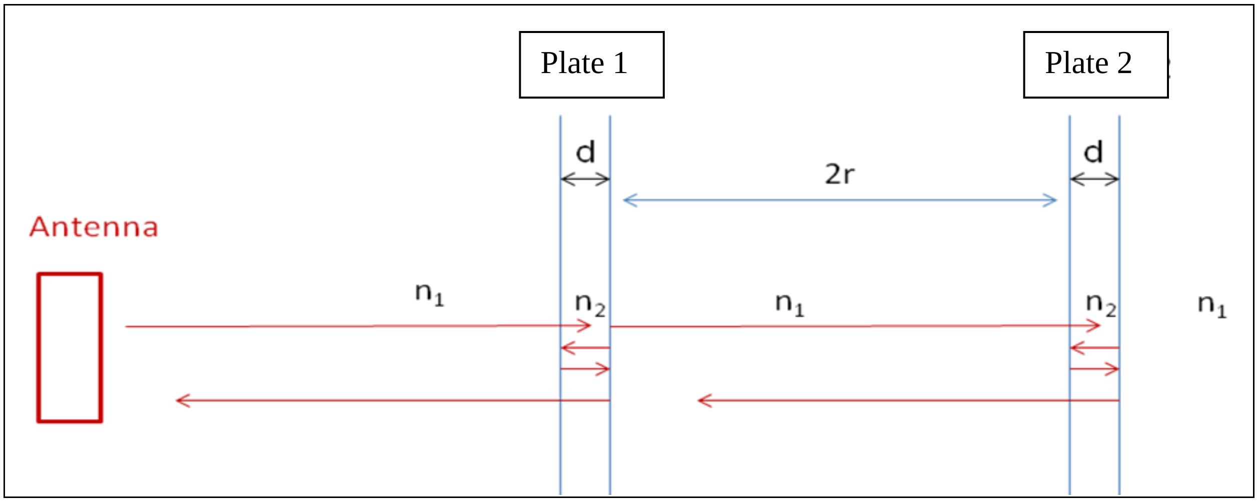

Figure 1. Simple model of a balloon being illuminated by electromagnetic radiation in the far-field.

[1] Andreasen, M. G. Back-scattering Cross Section of a Thin, Dielectric, Spherical Shell. *IEEE Transactions on Antennas and Propagation* **1957**, *5* (3), 267–270.

The RCS of each plate can be estimated using the high-frequency RCS approximation of a conducting sphere given by Ruck.[2] All the preceding calculations are scaled to the RCS calculated using equation 1. The reflection coefficient at the walls can be calculated using the Fresnel equations. For an E-field parallel or perpendicular to the plane of incidence at an angle of 0°, the parallel and perpendicular reflection coefficients are given by

$$\sigma = 4\pi r^2 \tag{1}$$

$$\rho_{\parallel} = -\rho_{\perp} = \frac{n_2 - n_1}{n_2 + n_1}\,. \tag{2}$$

The transmission coefficients for the parallel and perpendicular components of the E-field are given by Hecht and Zajac:[3]

$$t_{\parallel} = t_{\perp} = \frac{2n_1}{n_2 + n_1}\,. \tag{3}$$

In a thin film, multiple reflections occur between the boundaries, which are summed together to produce an E-field given by

$$E_r(\omega,t) = E_0 e^{j\omega t} p\left(\frac{p(1-e^{-j\omega 2d})}{1-p^2 e^{-j\omega 2d}}\right) \tag{4}$$

where E_0 is the initial electric field at the boundary of the plate, ω is the frequency, t is time, and j is an imaginary number.[4] Based upon the theory of superposition, the total electric field returned from the two films is

$$E_{tot}\left(\omega,t\right) = \left(E_{r1} + E_{r2} e^{-j\omega 4r}\right) e^{j\omega t} \tag{5}$$

Summing these two signals creates an interference pattern as a function of frequency. Since the signal reflected by the thin films is very small, multiple bounces between the wall 1 and wall 2 are ignored.

---

[2] Ruck G. T. et al. *Radar Cross Section Handbook - Volumes 1*; Plenun Press, New York, 1970.
[3] Hecht, E.; Zajac, A. *Optics*, Addison-Wesley, 1979, p. 75.
[4] Hecht, E.; Zajac, A. *Optics*, Addison-Wesley 1979, p. 304.

3. Simulation

The RCS of a balloon with a diameter of 1 m and a dielectric constant representing latex was calculated using equations 1–5 and results from a simulation based upon a paper by Andreasen.[1] The dielectric constant was assumed to be real and have no attenuation effects. Table 1 shows the values of the parameters used in the simulation. The code for the two plate model was written in Matlab and is listed in appendix A. The code for the model based upon Andreasen's paper was written in Mathematica and is listed in appendix B.

Table 1. Parameters used to simulate the RCS of a balloon.

Parameter	**Value**
Dielectric constant of air	1
Dielectric constant of the balloon	2.5
Radius of balloon	0.5 m
Mass of balloon	0.05 Kg
Density of balloon material	940 Kg/m^3
Speed of light	3e8 m/s

Figure 2 shows the estimated RCS of the balloon as a function of frequency, where "2 plate" in legend denotes the two plate model and "Andreasen" denotes the results calculated using Andreasen's method. The RCS calculated using the two models are in reasonable agreement and the frequency of the interference patterns matches closely. As the frequency is increased, the RCS trends higher. This is because the effective reflection coefficients for the two plates are smaller at lower frequencies. The multiple bounces in the dielectric film cancel better at lower frequencies. The RCS of a perfectly conducting sphere of the same size has a RCS of approximately –1 dBsm, which is much larger than the estimated RCS of the balloon.

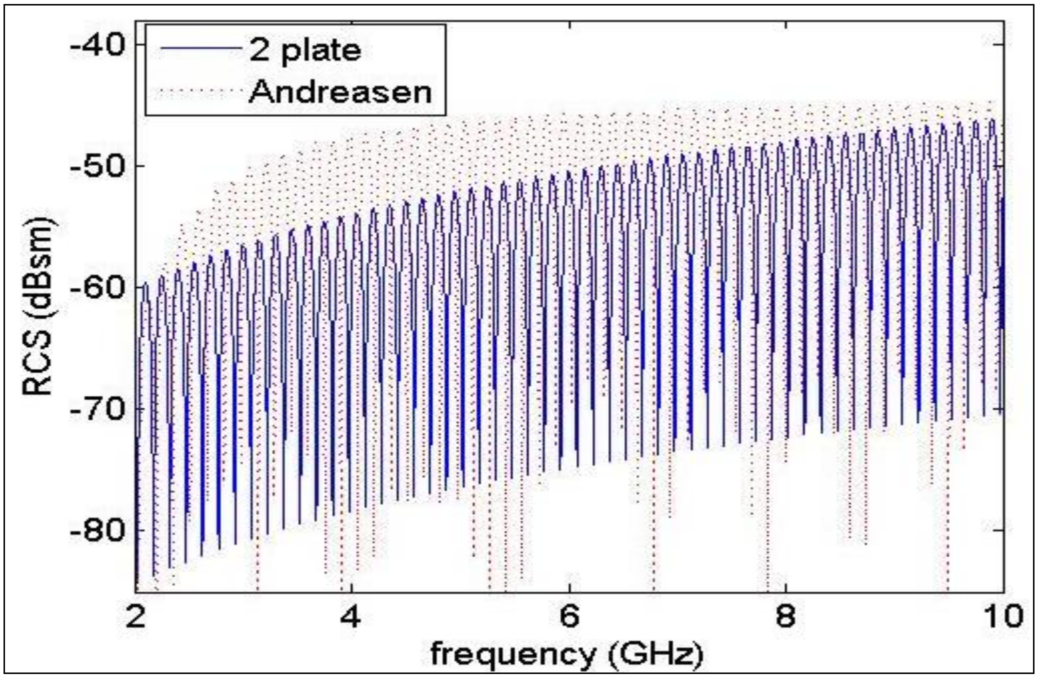

Figure 2. RCS of a balloon with a diameter of 1 m.

4. Conclusion

A model was developed to estimate the high-frequency RCS of a dielectric spherical shell. The model is based upon approximating the shell with two thin parallel dielectric plates. The model was used to estimate the RCS of a balloon with a 1-m diameter. The results were in reasonable agreement with a method developed by Andreasen. The RCS of a balloon with a 1-m diameter was determined to be much smaller than the RCS conducting sphere with the same dimensions for the frequencies of interest. For many scenarios, the RCS of a balloon can be considered negligible.

Appendix A. Matlab Code for the Two Plate Model

```
% matlab code to calculate the RCS of a balloon
% written by Geoffrey Goldman in September 2009
% modified August 2011

ep=2.5;      % epsilon for latex
nt=ep^0.5;   % latex index of refraction
ni=1;        % air
c=3e8;       % speed of light

mass=.05;  % kg
p=940;     % density
radius=0.5;  % radius of sphere (meters)

area=4*pi*radius^2; % approximate surface area of balloon

dr=mass/(p*area);    % approximate width of balloon

radius=radius-dr;    % radius of air sphere
area=4*pi*(radius+dr/2)^2; % surface area of balloon

dr=mass/(p*area);    % width of balloon

r_par=(nt-ni)/(nt+ni);   % 4.47, pg 75, E Hecht, A Zajac, Optics
r_per=-r_par;

t=2*ni/(ni + nt);  % both par and perpen 4.48 pg 75 ,E Hecht, A Zajac, Optics

tp_par=2*nt/(ni + nt);
tp_per=2*nt/(ni + nt);

t_tp=1-r_par^2;

E0 = (pi*radius^2)^0.5;

freq_array=(2:0.0025:10)*1e9; % calculate at these frequencies
[temp,Nfreq]=size(freq_array);

ifreq=0;

for freq1=1:Nfreq

    freq=freq_array(ifreq);
    lambda_air=c/freq;
    lambda_balloon=c/(freq*nt);
    phase_term=exp(-j*4*pi*dr/lambda_balloon);

    E0r_par=E0*r_par*(1-phase_term)/(1-phase_term*r_par^2);  % pg 305 , 9.29
b, E Hecht, A Zajac, Optics
```

```
    E0r_per=E0*r_per*(1-phase_term)/(1-phase_term*r_per^2);  % pg 305 , 9.29
b, E Hecht, A Zajac, Optics

    E0t_par=E0*t_tp/(1-phase_term*r_par^2);  % pg 305 , 9.29 b, E Hecht, A
Zajac, Optics
    E0t_per=E0*t_tp/(1-phase_term*r_per^2);  % pg 305 , 9.29 b, E Hecht, A
Zajac, Optics

    E0r_par=E0r_par + E0t_par^2*(E0r_par/E0)*exp(-j*radius*8*pi/lambda_air);
    E0r_per=E0r_per + E0t_per^2*(E0r_par/E0)*exp(-j*radius*8*pi/lambda_air);

    rcs_par(ifreq)=abs(E0r_par)^2;
    rcs_per(ifreq)=abs(E0r_per)^2;
end

load 'jeff.txt'  % load results from Andreasen model calculated by Frank
Crowne

figure
plot(freq_array/1e9,10*log10(rcs_par));
hold on
plot(jeff(:,1),jeff(:,2),'r')
%  plot(freq_array,10*log10(rcs_per),'r');
%  hold on
plot(freq_array/1e9,20*log10(E0*ones(1,Nfreq)),'m.-')
legend('model1','model2','conducting sphere')
ylabel('RCS (dBsm)')
xlabel('frequency (GHz)')
zoom on


f=figure
plot(freq_array/1e9,10*log10(rcs_par/E0^2));
hold on
plot(jeff(:,1),jeff(:,2),'r')
legend('2 plate','Andreasen')
ylab=ylabel('RCS (dBsm)')
xlab=xlabel('frequency (GHz)')

ca=get(f,'CurrentAxes')

set(gcf,'DefaultLineLineWidth',1.5);
set(xlab,'fontsize',14);
set(ylab,'fontsize',14);
set(ca,'fontsize',14);
axis([2 10 -80 -40])
```

Appendix B. Mathematica Code for the Model Based on Andreasen

```
Mathematica code to simulate the RCS of a balloon using a method developed by
Andreasen
Written by Frank Crowne, 2011

(* derivatives of terms in series *)
    D[x  SphericalBesselJ[n,x], x]
    1/2 (x SphericalBesselJ[?1+n,x] + SphericalBesselJ[n,x] ? x
SphericalBesselJ[1+n,x])
    D[x  SphericalHankelH2[n,x], x]
1/2 (x SphericalHankelH2[?1+n,x] + SphericalHankelH2[n,x] ? x
SphericalHankelH2[1+n,x])
    (* assembling series terms *)
    R[n_,x1_,s_]:=
(F1=1/2 (x1 SphericalBesselJ[?1+n,x1] + SphericalBesselJ[n,x1] ? x1
SphericalBesselJ[1+n,x1]);
    G1=1/2 (x1 SphericalHankelH2[?1+n,x1] + SphericalHankelH2[n,x1] ?x1
SphericalHankelH2[1+n,x1]);
    1/(1+I s F1 G1))
(* evaluate R[4,1.,.6] as a check *)
R[4,1.,.6]
    0.435717-2.80521*10^-6 I
S[n_,x1_,s_]:=
( F1=x1 SphericalBesselJ[n,x1];
G1=x1 SphericalHankelH2[n,x1];
1/(1+I s F1 G1) )
    (* complete n-th series term expressed as Mathematica function *)
    SERTERM[n_,x1_,s_]:=
( F1=1/2 (x1 SphericalBesselJ[?1+n,x1] + SphericalBesselJ[n,x1] ? x1
SphericalBesselJ[1+n,x1]);
    F2=x1 SphericalBesselJ[n,x1];
(?1)^n (2n+1)(F1^2 R[n, x1, s] ? F2^2 S[n, x1, s]) )
    (* calculate series for RCS *)
    RCS[x1_,s_]:=(s/x1)^2 Abs[Sum[SERTERM[n, x1, s],{n,10}]]^2
    RCS[.3,5.]
    0.00576843

    Plot[ RCS[x1,.4],{x1,0.,5.}]
```

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